TEMA 2.- GEOMETRÍA DEL ELIPSOIDE DE REVOLUCIÓN

EJERCICIO 2.1

$a = 5; \quad c = 3 \qquad a^2 = b^2 + c^2 \quad \Rightarrow \quad b = \sqrt{a^2 - c^2} = \sqrt{25 - 9} = 4$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$

EJERCICIO 2.2

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \quad \Rightarrow \quad \boxed{\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{16} = 1}$$

$$f = \frac{a-b}{a} = \left\{ \begin{matrix} a = 5 \\ b = 4 \end{matrix} \right\} = \frac{5-4}{5} \quad \Rightarrow \quad \boxed{f = 1/5 = 0{,}2}$$

$$e = \frac{c}{a} = \frac{3}{5} \quad \Rightarrow \quad \boxed{e = 3/5 = 0{,}6}; \qquad e' = \frac{c}{b} = \frac{3}{4} \quad \Rightarrow \quad \boxed{e' = 3/4 = 0{,}75}$$

EJERCICIO 2.3

$$f = 1 - \frac{b}{a} \quad \Rightarrow \quad \boxed{b = a \cdot (1 - f)}$$

$$c = \sqrt{a^2 - b^2} = \sqrt{a^2 - a^2 (1-f)^2} = a\sqrt{1 - (1-f)^2} = a\sqrt{1 - 1 - f^2 + 2f} \quad \Rightarrow \quad \boxed{c = a\sqrt{f(2-f)}}$$

$$e = \frac{c}{a} = \frac{a\sqrt{f(2-f)}}{a} \quad \Rightarrow \quad \boxed{e = \sqrt{f(2-f)}}; \qquad e' = \frac{c}{b} = \frac{a\sqrt{f(2-f)}}{a(1-f)} \quad \Rightarrow \quad \boxed{e' = \frac{\sqrt{f(2-f)}}{1-f}}$$

$$\left. \begin{matrix} a = 6.378.388 \ m \\ f = 1/297 \end{matrix} \right\} \qquad \boxed{b = 6.356.911{,}95 \ m} \quad \boxed{c = 522.976{,}09 \ m}$$

$$\boxed{e = 0{,}08199188998} \quad \boxed{e' = 0{,}08226888961}$$

EJERCICIO 2.4

$$\left. \begin{matrix} a = 6.378.137 \ m \\ f = 1/298{,}257 \end{matrix} \right\} \qquad b = a(1-f) \quad \Rightarrow \quad \boxed{b = 6.356.752{,}30 \ m}$$

$$c = a\sqrt{f(2-f)} \quad \Rightarrow \quad \boxed{c = 521.854{,}20 \ m}$$

$$e = \sqrt{f(2-f)} \quad \Rightarrow \quad \boxed{e = 0{,}08181922146}$$

$$e' = \frac{\sqrt{f(2-f)}}{1-f} \quad \Rightarrow \quad \boxed{e' = 0{,}08209446887}$$

EJERCICIO 2.5

$$N = \frac{a}{w} = \frac{a}{\sqrt{1-e^2 sen^2\varphi}} \quad \Rightarrow \quad \boxed{N = \frac{a}{\sqrt{1-f(2-f)\,sen^2\varphi}}}$$

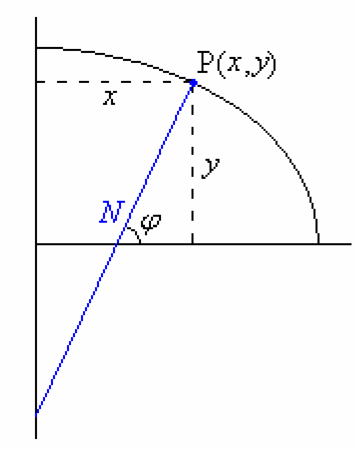
Internacional.-

$$N = \frac{a}{\sqrt{1-f(2-f)\,sen^2\varphi}} = \frac{6.378.388\ m}{\sqrt{1-\frac{1}{297}\left(2-\frac{1}{297}\right)sen^2\,30^o}} = \frac{6.378.388\ m}{0{,}999159312} \quad \Rightarrow \quad \boxed{N = 6.383.754{,}74\ m}$$

WGS84.-

$$N = \frac{a}{\sqrt{1-f(2-f)\,sen^2\varphi}} = \frac{6.378.137\ m}{\sqrt{1-\frac{1}{298{,}257}\left(2-\frac{1}{298{,}257}\right)sen^2\,30^o}} = \frac{6.378.137\ m}{0{,}999162851} \quad \Rightarrow \quad \boxed{N = 6.383.480{,}92\ m}$$

EJERCICIO 2.6



$$x = N\cos\varphi$$
$$y = N\left(1-e^2\right)sen\varphi$$

$$\left.\begin{array}{l} Hayford \\ a = 6.378.388\ m \\ f = 1/297 \end{array}\right\} \quad \varphi = 30^o$$

$$w = \sqrt{1-f\,(2-f)\,sen^2\varphi} \quad \Rightarrow \quad w = 0{,}999159312$$

$$N = \frac{a}{w} = \frac{6.378.388\ m}{0{,}999159312} \quad \Rightarrow \quad N = 6.383.754{,}74\ m$$

$$x = N\cos\varphi = 6.378.754{,}74\cos 30^o \quad \Rightarrow \quad \boxed{x = 5.528493{,}78\ m}$$

$$y = N\left(1-e^2\right)sen\varphi = N(1-f(2-f))sen\varphi = 6.378.754{,}74\left(1-\frac{1}{297}\left(2-\frac{1}{297}\right)\right)sen30^o \Rightarrow \boxed{y = 3.170.419{,}43\ m}$$

EJERCICIO 2.7

$$\rho = \frac{a\left(1-e^2\right)}{w^3} = \frac{a\left(1-f(2-f)\right)}{\left(1-e^2 sen^2\varphi\right)^{3/2}} \quad \Rightarrow \quad \boxed{\rho = \frac{a\left(1-f(2-f)\right)}{\left(1-e^2 sen^2\varphi\right)^{3/2}}} \quad \boxed{N = \frac{a}{\sqrt{1-f(2-f)\,sen^2\varphi}}}$$

$$\left.\begin{array}{l} a = 6.378.137\ m \\ f = 1/298{,}257 \end{array}\right\} \qquad \boxed{\rho = 6.351.377{,}08\ m} \qquad \boxed{N = 6.383.480{,}92\ m}$$

EJERCICIO 2.8

$$R_\alpha = \frac{N\,\rho}{\rho\,sen^2\alpha + N\cos^2\alpha}$$

$$\rho = \frac{a\left(1-f(2-f)\right)}{\left(1-e^2 sen^2\varphi\right)^{3/2}} = \frac{6.335.508{,}202\ m}{0{,}9949622369} \quad \Rightarrow \quad \boxed{\rho = 6.367.586{,}60\ m}$$

$$N = \frac{a}{\sqrt{1-f(2-f)\,sen^2\varphi}} = \frac{6.378.388\ m}{0{,}9983179178} \quad \Rightarrow \quad \boxed{N = 6.389.135{,}05\ m}$$

$$R_\alpha = \frac{N\,\rho}{\rho\,sen^2\alpha + N\cos^2\alpha} \quad \Rightarrow \quad R_{20} = \frac{N\,\rho}{\rho\,sen^2 20^o + N\cos^2 20^o} \quad \Rightarrow \quad \boxed{R_{20} = 6.370.099{,}78\ m}$$

En efecto es: $\boxed{\rho < R_\alpha < N}$

EJERCICIO 2.9

$$w = \sqrt{1-f\,(2-f)\,sen^2\varphi} \quad \Rightarrow \quad w = 0{,}998500217 \quad \Rightarrow \quad w^3 = 0{,}995507395$$

$$\rho = \frac{a\left(1-e^2\right)}{w^3} = \frac{a\left(1-f(2-f)\right)}{w^3} = \frac{6.335.439{,}295\ m}{0{,}995507395} \quad \Rightarrow \quad \rho = 6.364.030{,}37\ m$$

$$N = \frac{a}{w} = \frac{6.378.137\ m}{0{,}998500217} \quad \Rightarrow \quad N = 6.387.717{,}19\ m$$

$$R = \sqrt{N\cdot\rho} \quad \Rightarrow \quad \boxed{R = 6.375.862{,}78\ m}$$

EJERCICIO 2.10

$$\left.\begin{array}{l} a = 6.378.388\ m \\ f = 1/297 \end{array}\right\} \qquad w = \sqrt{1-f\,(2-f)\,sen^2\varphi} \quad \Rightarrow \quad \left\{\begin{array}{l} w_{36} = 0{,}998838012 \\ w_{43} = 0{,}998435345 \end{array}\right.$$

$$N_{36} = \frac{a}{w_{36}} = \frac{6.378.388\ m}{0{,}998838012} = 6.385.808{,}233\ m\ ; \qquad N_{43} = \frac{a}{w_{43}} = \frac{6.378.388\ m}{0{,}998435345} = 6.388.383{,}61\ m$$

$$S_p = N \cos\varphi \, \Delta\lambda$$

$$S_{p\,36} = N_{36}\left(\cos 36^o\right)\frac{\pi}{3600 \cdot 180} \quad \Rightarrow \quad \boxed{S_{p\,36} = 25{,}05\ m}$$

$$S_{p43} = N_{43}\left(\cos 43^o\right)\frac{\pi}{3600 \cdot 180} \quad \Rightarrow \quad \boxed{S_{p\,43} = 22{,}65\ m}$$

EJERCICIO 2.11

$$S_m = \rho_m \Delta\varphi\left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right]$$

$$\left.\begin{array}{l} a = 6.378.388\ m \\ f = 1/297 \end{array}\right\} \qquad e^2 = f(2-f) \quad \Rightarrow \quad e^2 = 0{,}006722670022$$

$$w^3 = \left(1 - e^2\ sen^2\varphi\right)^{3/2} \quad \Rightarrow \quad \begin{cases} w_{36}^3 = 0{,}996518085 \\ w_{43}^3 = 0{,}995313378 \end{cases}$$

$$\rho_m = \frac{a\left(1-e^2\right)}{w^3} = \frac{a\left(1 - f(2-f)\right)}{w^3} \quad \Rightarrow \quad \begin{cases} \rho_{m\,36} = 6.357.644{,}98\ m \\ \rho_{m\,43} = 6.365.340{,}15\ m \end{cases}$$

$$S_{m\,36} = \rho_{m\,36}\ \Delta\varphi\left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right] = 6.357.644{,}98 \cdot \frac{\pi}{3600 \cdot 180}\left[1 + \frac{0{,}006722670022}{8}\left(\frac{\pi}{3600 \cdot 180}\right)^2 \cos 72^o\right]$$

$$S_{m\,43} = \rho_{m\,43}\ \Delta\varphi\left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right] = 6.365.340{,}15 \cdot \frac{\pi}{3600 \cdot 180}\left[1 + \frac{0{,}006722670022}{8}\left(\frac{\pi}{3600 \cdot 180}\right)^2 \cos 86^o\right]$$

$$S_{m\,36} = \rho_{m\,36}\Delta\varphi\left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right] = 30{,}823 \cdot \left[1 + 1{,}38 \cdot 10^{-15}\right] \quad \Rightarrow \quad \boxed{S_{m\,36} = 30{,}823\ m}$$

$$S_{m\,43} = \rho_{m\,43}\Delta\varphi\left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right] = 30{,}860 \cdot \left[1 + 1{,}38 \cdot 10^{-15}\right] \quad \Rightarrow \quad \boxed{S_{m\,43} = 30{,}860\ m}$$

EJERCICIO 2.12

$$S_m = \rho_m \Delta\varphi\left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right]$$

$$e^2 = f(2-f) \quad \Rightarrow \quad e^2 = 0{,}006694385 \qquad ; \qquad w_5^3 = \left(1 - e^2\ sen^2\,5^o\right)^{3/2} \quad \Rightarrow \quad w_5^3 = 0{,}999923723$$

$$\varphi_m = 5^o\,; \qquad \Rightarrow \qquad \rho_m = \frac{a\left(1-e^2\right)}{w_5^3} = \frac{a\left(1-f(2-f)\right)}{w_5^3} \qquad \Rightarrow \qquad \rho_m = 6.335.922{,}575\ m$$

$$S_m = \rho_m \Delta\varphi \left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right] = 6.335.922{,}575 \cdot 10\frac{\pi}{180}\left[1 + \frac{0{,}006694385}{8}\left(\frac{10\,\pi}{180}\right)^2 \cos 10^o\right]$$

$$\boxed{S_m = 1.105.854{,}86\ m}$$

EJERCICIO 2.13

$$e^2 = f(2-f) \quad \Rightarrow \quad e^2 = 0{,}006694385 \quad ; \quad w_{15}^3 = \left(1 - e^2\ sen^2\, 15^o\right)^{3/2} \quad \Rightarrow \quad w_{15}^3 = 0{,}999327417$$

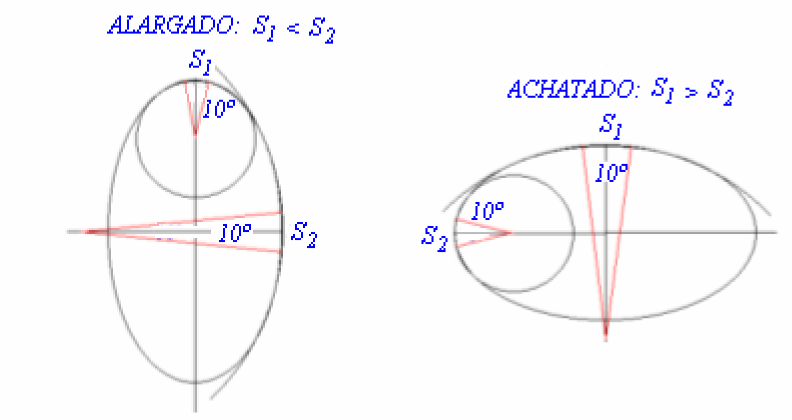
$$\varphi_m = 15^o\,; \qquad \Rightarrow \qquad \rho_m = \frac{a\left(1-e^2\right)}{w_{15}^3} = \frac{a\left(1-f(2-f)\right)}{w_{15}^3} \qquad \Rightarrow \qquad \rho_m = 6.339.703{,}27\ m$$

$$S_m = \rho_m \Delta\varphi \left[1 + \frac{e^2}{8}\Delta\varphi^2 \cos 2\varphi_M\right] = 6.339.703{,}27 \cdot 10\frac{\pi}{180}\left[1 + \frac{0{,}006694385}{8}\left(\frac{10\,\pi}{180}\right)^2 \cos 30^o\right]$$

$$\boxed{S_m = 1.106.512{,}29\ m}$$

La longitud de este arco es mayor que la del ejercicio anterior, es decir:

$$S_{0^o-10^o} < S_{10^o-20^o}$$



Se trata del caso de la derecha de la figura, en el que el arco medido a mayor latitud da mayor longitud, por tanto, se demuestra que en los dos últimos ejercicios se trabaja con un elipsoide achatado por los polos.